FYS-STK4155, OCT 21, 2022

Differential equations

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

initial conditions

$$u(x,y,t_0) = u(x,y,0) = g(x,y)$$

$$t_0 = 0$$

Boundary conditions

$$u(x_0,y,t) = f(y,t)$$

$$u(x_m,y,t) = h(y,t) \quad \ldots \text{ etc}$$

We remote $\frac{\partial u}{\partial t} = D(\ldots)$

$$F\left(\vec{x},\vec{y},t,u(x,y,t),\frac{\partial u}{\partial t},\nabla_x u,\nabla_y u,\nabla_x^2 u,\nabla_y^2 u\right) = 0$$

$$F = \frac{\partial u}{\partial t} - D\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right\}$$

ansatz:

$$u(x,y,t) = g_0(x,y,t) + g\left(x,y,t, NN(x,y,t;\theta)\right)$$

parameters.

Typical cost/loss function

$$C(x,y,t;\theta) = \| F(x,y,t;\theta) \|_2^2$$

$$\hat{\theta} = \underset{\theta \in \mathbb{R}}{\operatorname{argmin}} \; C(x,y,t;\theta)$$

Example:

$$g(0) = 0 \qquad g(1) = 0$$

$$x \in [0,1]$$

$$g(x) = g_0 + x(1-x)\,N(x;\theta)$$

Convolution & NN (CNN)

Sound file:

8000 samples per second

run for 2 minutes

Classification problem of the type true/False.

$X \sim \mathbb{R}^{10^6}$

NN with 3 layers

10^4 nodes in layer 1, n_1

$W_1 \in \mathbb{R}^{10^6 \times 10^4}$ → 10^{10} weights to optimize

parameters $\sim 10^{10} + 10^4 \sim 10^{10}$

Layer 2

$n_2 = 10^3$

$W_2 \in \mathbb{R}^{10^4 \times 10^3}$

we have $10^7 + 10^3$ parameters

$\approx 10^7$

Layer 3 = output layer

only one node $\Rightarrow$

10^3 parameters

Adding up : $10^{10} + 10^9 + 10^7$

$+ 10^3 + 10^3 + 1 \approx 10^{10}$