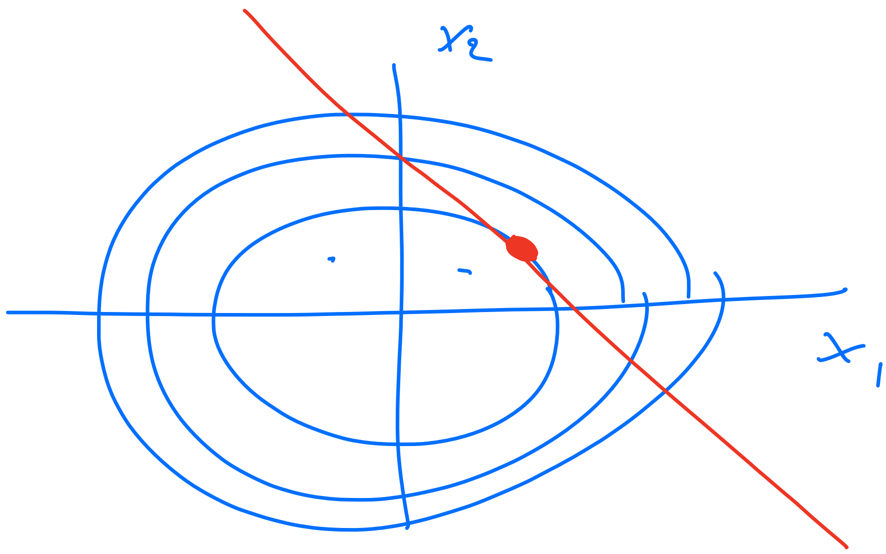
Lecture November 26

Example

$$f(x) = 1 - x_1^2 - x_2^2$$



$$g(x) = x_1 + x_2 - 1 = 0$$

$$\mathcal{L}(x, \lambda) = f(x) + \lambda g(x)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{\partial \mathcal{L}}{\partial x_2} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow x_1 + x_2 = 1$$

$g(x) > 0$ $\qquad$ $\nabla f(x) = 0$

$\lambda = 0$

$\lambda g(x) > 0$

on the boundary $g(x) = 0$

$\lambda \neq 0$

$\lambda \cdot g(x) = 0$

Minimization

$$\mathcal{L}(x, \lambda) = f(x) - \lambda g(x)$$

$$g(x) \geq 0$$

$$\lambda g(x) = 0$$

$$\lambda \geq 0$$

Karush-Kuhn-Tucker conditions

$$D = \{(x_0, y_0), (x_1, y_1) \ldots (x_{n-1}, y_{n-1})\}$$

$$f(x) = w^T x + b$$

$$y_i f(x_i) = y_i (w^T x_i + b) \geq 1$$

$$\mathcal{L}(w, b, \lambda) = \frac{1}{2} w^T w -$$

$$\sum_{i=0}^{n-1} \lambda_i \left(y_i (w^T x_i + b) - 1 \right)$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 = - \sum_i \lambda_i y_i = 0$$

$$\frac{\partial \mathcal{L}}{\partial w} = 0 = w - \sum_i \lambda_i y_i x_i$$

$$y_i = \{-1, +1\}$$

$$\mathcal{L} = \sum_i \lambda_i - \frac{1}{2} \sum_{i,j} \lambda_i \lambda_j y_i y_j x_i^T x_j$$

subject to $\lambda \geq 0$ and

$$\lambda_i \left[y_i (w^T x_i + b) - 1 \right] = 0$$

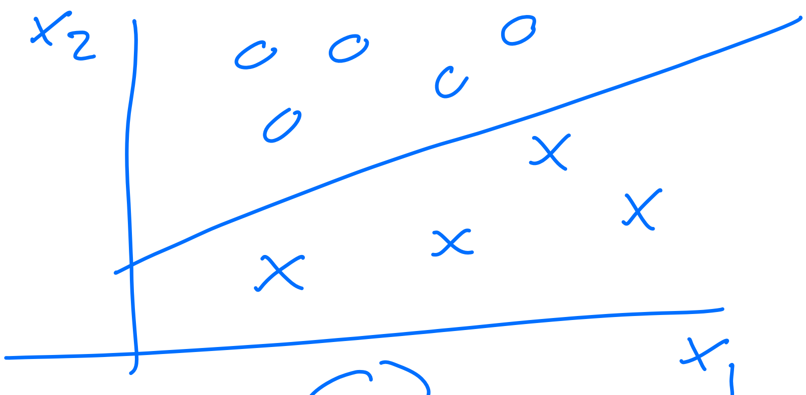
$$y_i (w^T x_i + b) - 1 \geq 0$$

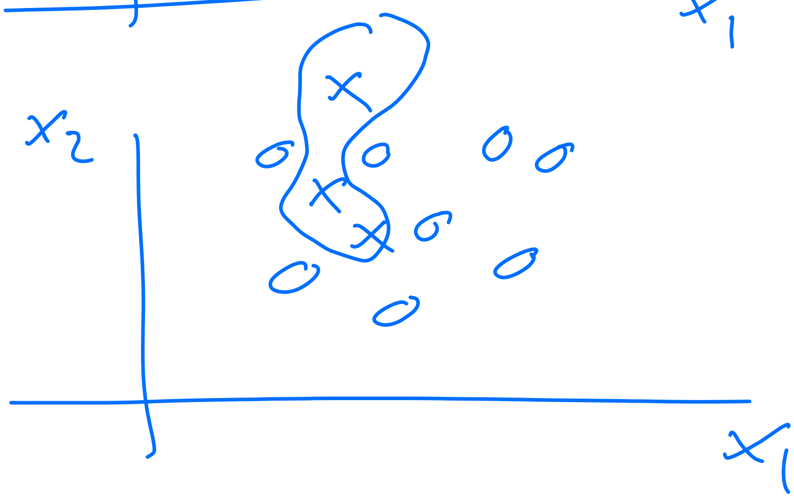
solve wrt λ_i

$$w = \sum_i \lambda_i y_i x_i$$

$$f(x) = w^T \Phi(x) + b$$

Kernel transformation (motivation)





$$f(x) = w^T z + b$$

$$(z = \phi(x))$$

$$= \left(\sum \lambda_i y_i x_i\right)^T \phi(x) + b$$

$$\phi(x_i)$$

$$= \sum_i \left(\lambda_i\, y_i\, \phi(x_i)\right)^T \phi(x) + b$$

$$\phi(x) = (1, x_1, x_2, x_1 x_2, x_1^2, x_2^2)$$

kernel $K(x, x') = \phi(x)^T \phi(x')$