FYS-STK3155/4155, October 14, 2024

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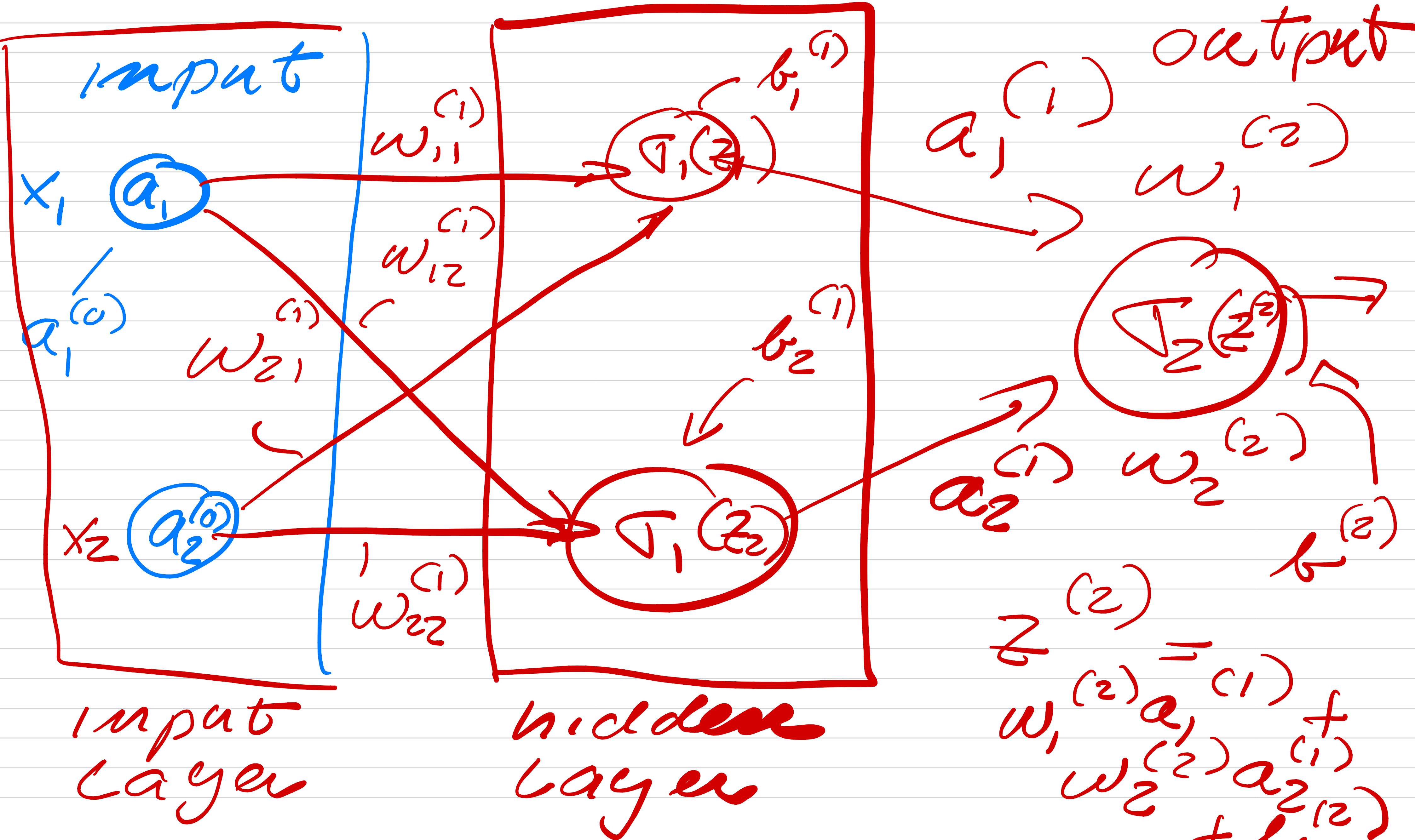

$$z^{(2)} = w_1^{(2)} a_1^{(1)} + w_2^{(2)} a_2^{(1)} + b^{(2)}$$

$$a^{(2)} \longrightarrow \boxed{c} \longleftarrow y$$

$$\Theta = \left\{ w_{11}^{(1)},\ w_{12}^{(1)},\ w_{21}^{(1)},\ w_{22}^{(1)},\ b_1^{(1)},\ b_2^{(1)},\ w_1^{(2)},\ w_2^{(2)},\ b^{(2)} \right\}$$

$$\begin{bmatrix} z_1^{(1)} \\ z_2^{(1)} \end{bmatrix} = \begin{bmatrix} w_{11}^{(1)} & w_{12}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} \end{bmatrix}^T \begin{bmatrix} a_1^{(0)} \\ a_2^{(0)} \end{bmatrix} + \begin{bmatrix} b_1^{(1)} \\ b_2^{(1)} \end{bmatrix}$$

$$z_1^{(1)} = w_{11}^{(1)} a_1^{(0)} + w_{21}^{(1)} a_2^{(0)} + b_1^{(1)}$$

$$z_j^{(l)} = \sum_{i=1}^{M_{l-1}} w_{ij}^{(l)} a_i^{(l-1)} + b_j^{(l)}$$

$$\begin{bmatrix} a_1^{(1)} \\ a_2^{(1)} \end{bmatrix} = \begin{bmatrix} \sigma_1(z_1^{(1)}) \\ \sigma_1(z_2^{(1)}) \end{bmatrix}$$

$$z^{(2)} = w_1^{(2)} a_1^{(1)} + w_2^{(2)} a_2^{(1)} + b^{(2)}$$

$$a^{(2)} = \sigma_2(z^{(2)})$$

$$\frac{\partial C}{\partial w_1^{(2)}} = \frac{\partial C}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\underbrace{\partial w_1^{(2)}}_{a_1^{(1)}}}$$

$$C = \frac{1}{2}\left(a^{(2)} - y\right)^2$$

$$\frac{\partial C}{\partial a^{(2)}} = \left(a^{(2)} - y\right)$$

$$\frac{\partial a^{(2)}}{\partial z^{(2)}} = \frac{\partial \sigma_2^{(2)}}{\partial z^{(2)}} = \sigma_2'$$

$$\frac{\partial C}{\partial w_1^{(2)}} = \left(a^{(2)} - y\right)\sigma_2' \, a_1^{(1)}$$

$$\delta^{(2)} = \frac{\partial C}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial z^{(2)}}$$

$$\frac{\partial C}{\partial b^{(2)}} = \frac{\partial C}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial z^{(2)}} \underbrace{\frac{\partial z^{(2)}}{\partial b^{(2)}}}_{= 1}$$

$$= \delta^{(2)}$$

$$\frac{\partial C}{\partial w_{ij}^{(1)}} \wedge \frac{\partial C}{\partial b_i^{(1)}}$$

$$\frac{\partial C}{\partial w_{ij}^{(1)}} = \underbrace{\frac{\partial C}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial z^{(2)}}}_{\delta^{(2)}} \frac{\partial z^{(2)}}{\partial z_1^{(1)}} \frac{\partial z_1^{(1)}}{\partial w_{11}^{(1)}}$$

$$\boxed{i = 1 \quad j = 1}$$

$$z_1^{(1)} = w_{11}^{(1)} a_1^{(0)} + w_{21}^{(1)} a_2^{(0)} + b_1^{(1)}$$

$$\frac{\partial z^{(2)}}{\partial z_1^{(1)}} \frac{\partial z_1^{(1)}}{\partial w_{11}^{(1)}} =$$

$$\frac{\partial z^{(2)}}{\partial a_1^{(1)}} \frac{\partial a_1^{(1)}}{\partial z_1^{(1)}}$$

$$\frac{\partial C}{\partial w_{11}^{(1)}} = \underbrace{\delta_1^{(1)}} a_1^{(1)}$$

$$\delta^{(2)} w_1^{(2)} \sigma'_1 = \delta_1^{(1)}$$

$$\Theta = \{ \Theta^{L}, \Theta^{L-1}, \Theta^{L-2}, \ldots, \Theta^{1} \}$$

$$\Theta^{L} = \{ w^{L}, b^{L} \}$$

$$z_j^{l} = \sum_{i=1}^{M_{l-1}} w_{ij}^{l} \, a_{l}^{l-1} + b_j^{l}$$

$$z^{l} \quad (w^{l})^{T} a^{l-1} + b^{l}$$

$$a_j^{l} = \sigma(z_j^{l}) = \frac{1}{1+\exp(-z_j^{l})}$$

$$\frac{\partial z_j^l}{\partial w_{ij}^l} = a_i^{l-1}$$

$$\frac{\partial z_j^l}{\partial a_i^{l-1}} = w_{ij}^l$$

$$\frac{\partial a_j^l}{\partial z_j^l} = \sigma'(z_j^l) = a_j^l(1-a_j^l)$$

$$= \sigma(z_j^l)(1-\sigma(z_j^l))$$

$l = L$ output layer

$$C(\theta^L) = \frac{1}{2}\sum_{i=1}^{n}\left(a_i^L(\theta^L) - y_i\right)^2$$

$$\frac{\partial C}{\partial w_{ij}^L} = (a_j^L - y_j)\frac{\partial a_j^L}{\partial w_{ij}^L}$$

$$\frac{\partial a_j^L}{\partial w_{ij}^L} = \frac{\partial a_j^L}{\partial z_j^L}\,\frac{\partial z_j^L}{\partial w_{ij}^L}$$

$$\delta_j^L = \underbrace{a_j^L(1-a_j^L)}_{\sigma'(z_j^L)}\underbrace{(a_j^L - y_j)}_{\frac{\partial C}{\partial a_j^L}}$$

$$\frac{\partial z_j^L}{\partial w_{ij}^L} = a_i^{L-1}$$

$$\frac{\partial C}{\partial w_{ij}^L} = \delta_j^L \cdot a_i^{L-1}$$

Hadamard multi

$$\delta^L = \sigma'(z^L) \odot \frac{\partial C}{\partial a^L}$$

$$\frac{\partial C}{\partial b_j^L} = \underbrace{\frac{\partial C}{\partial a_j^L} \frac{\partial a_j^L}{\partial z_j^L}}_{\delta_j^L} \underbrace{\frac{\partial z_j^L}{\partial b_j^L}}_{=1}$$

$$= \delta_j^L$$

$$w_{ij}^L \leftarrow w_{ij}^L - \eta \, \delta_j^L a_i^{L-1}$$

$$b_j^L \leftarrow b_j^L - \eta \, \delta_j^L$$

$\delta_j^l = ?$ we want to express in terms of the equations for layer $l+1$

$$\delta^L = \frac{\partial C}{\partial a^L} \frac{\partial a^L}{\partial z^L} = \frac{\partial C}{\partial z^L}$$

$$\delta_j^L = \frac{\partial C}{\partial z_j^L}$$ want δ_j^l

$$\delta_j^l = \frac{\partial C}{\partial z_j^l} = \sum_k \frac{\partial C}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial z_j^l}$$

$$z_j^{l+1} = \sum_{i=1}^{M_l} w_{ij}^{l+1} a_i^l + b_j^{l+1}$$

$$\Rightarrow \delta_j^l = \sum_k w_{ij}^{l+1} \delta_k^{l+1} \cdot \sigma'(z_j^l)$$

$$\sigma(z_j^l)$$

$$w_{ij}^l \Leftarrow w_{ij}^l - \eta \delta_j^l a_i^{l-1}$$

$$b_j^l \Leftarrow b_j^l - \eta \delta_j^l$$