Lecture FYS-STK3155/4155, October 26, 2023

ordinary differential eqs (ODEs)

1st order

$$\frac{df}{dx} = g(x)$$

initial condition $f(x=0) = a$

$f(x) \Rightarrow f(x_i) = f_i$

$x \Rightarrow x_i = \{x_0, x_1, \ldots x_n\}$

$i = 0, 1, 2, \ldots n$ ($n+1$ points)

$f(x=0) = f(x_0) = f_0$

$x_i = x_0 + i\,\Delta x \qquad \Delta x = \dfrac{x_n - x_0}{n}$

Taylor expand $f(x)$ around $f(x+\Delta x)$ or $f(x-\Delta x)$

$$f(x+\Delta x) = f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2!} f'' + O((\Delta x)^3)$$

$$f(x+\Delta x) - f(x) \simeq \Delta x f'(x)$$

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = f_i' = \frac{f(x_i+\Delta x) - f(x_i)}{\Delta x}$$

$$O(\Delta x) = (f_{i+1} - f_i)/\Delta x$$

$$f_i'' = \frac{f_{i+1} + f_{i-1} - 2f_i}{(\Delta x)^2} \qquad (O(\Delta x^2))$$

$$f_i' = \frac{f_{i+1} - f_{i-1}}{2\Delta x} \qquad (O(\Delta x^2))$$

$$\frac{df}{dx} = g(x)$$

$$\frac{f_{i+1} - f_i}{\Delta x} = g_i \Rightarrow$$

$$f_{i+1} = f_i + \Delta x\, g_i$$

$$\frac{d^2 f}{dx^2} = -\alpha^2 x$$

$$f(x) = A \cdot \cos(\alpha x) + B \sin(\alpha x)$$

$$f(x=0) = f(x_0) = f_0 = 0 \Rightarrow$$

$$A = 0$$

Neural network strategy:

$$\frac{df}{dx} = g(x) \Rightarrow$$

$$\left(\frac{df}{dx} - g(x) \right) = 0$$

start with guess

$$h(x) = h_0 + NN(x)$$

obeys initial conditions

$$\boxed{g'(x) = -\gamma g(x)}$$

$$g_t(x) = g_0 + x N(x, P)$$

$$g_t(x=0) = g_0 \,,$$ initial conditions

$$g_t'(x) = x \frac{dN}{dx} + N(x, P)$$