FYS-STK 4155, Oct 20, 2022

NNs and ordinary diff eqs (ODEs)

1st-order:

$$\frac{dy(x)}{dx} = f(x)$$

initial conditions

$$f(x_0) = f_0$$

$$x \in [x_0, x_n]$$

$$x \rightarrow x_i = x_0 + \Delta x\, i$$

$$i = 0, 1, 2, \ldots n$$

$$\Delta x = \frac{x_n - x_0}{n}$$

$$y \rightarrow y(x_i) = y_i$$

$$f \rightarrow f(x_i) = f_i$$

Taylor expand y around $x_i \pm \Delta x$

$$y(x_i \pm \Delta x) = y(x_i) \pm \Delta x\, y'(x_i) + \frac{\Delta x^2}{2!} y''(x_i) + O(\Delta x^3)$$

$$y(x_i + \Delta x) = y_{i+1} \simeq y_i + \Delta x\, y_i'$$

$$y_i' = f(x_i) = f_i \Rightarrow$$

$$y_{i+1} = y_i + \Delta x\, f_i$$

Forward Euler.

Neural Network solution:

$$\frac{dy}{dx} = f(x)$$

Basic philosophy is to compute a cost function which is the difference

$$\left\| \frac{dy}{dx} - f(x) \right\|_2^2$$

$$y(x_i) \longrightarrow \tilde{y}_0 + \underbrace{NN(x_i;\theta)}_{\text{neural network}}$$

initial conditions