# Lecture September 2

$X \in \mathbb{R}^{n \times p}$

SVD theorem

$X = U \Sigma V^T$

$U \in \mathbb{R}^{n \times n}$ $\quad UU^T = U^T U = \mathbb{1}$

$\Sigma \in \mathbb{R}^{n \times p}$

$n \geq p$

$\left\{ X^T X \in \mathbb{R}^{p \times p} \right\}$

$p \ll n$

$V \in \mathbb{R}^{p \times p}$ $\quad VV^T = V^T V = \mathbb{1}$

$\Sigma$ contains only diagonal values

$\Sigma = \begin{bmatrix} \sigma_1 & & \end{bmatrix}$

$$\Sigma = \begin{bmatrix} & \sigma_2 & & & & \\ & & \ddots & & & \\ & & & \sigma_p & & \\ & & & & 0 & \\ & & & & & 0 \quad 0 \end{bmatrix}$$

$$\sigma_1 > \sigma_2 > \sigma_3 \ldots > \sigma_p > 0$$

Mathematical properties of OLS & Ridge Regression.

- $\hat{\beta}_{OLS} = (X^T X)^{-1} X^T y$

$$\tilde{y} = X (X^T X)^{-1} X^T y$$

$$= X \hat{\beta}_{OLS}$$

$$A = X (X^T X)^{-1} X^T$$

$$A^2 = A$$

projection matrix

Example

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \ ? \ = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$$\boxed{X^T X = \mathbb{1} = X X^T}$$

$$\hat{y}_{ols} = X \hat{\beta}_{ols} =$$

$$A y$$

$$= X (X^T X)^{-1} X^T y$$

$$= y$$