FYS-STK3155/4155 lecture August 25, 2025

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$$y^T = [y_0\ y_1\ \dots\ y_{n-1}]$$

$$y \in \mathbb{R}^n$$

$$x^T = [x_0\ x_1\ \dots\ x_{n-1}]$$

$$x \in \mathbb{R}^n$$

$$\theta^T = [\theta_0\ \theta_1\ \dots\ \theta_{p-1}]$$

$$\theta \in \mathbb{R}^p$$

$$\tilde{y}^T = [\tilde{y}_0 \; - - - \; \tilde{y}_{n-1}]$$

$$\tilde{y}_n = \sum_{j=0}^{P-1} \theta_j x_n^j$$

$$= \theta_0 x_n^0 + \theta_1 x_n^1 + \theta_2 x_n^1 + \cdots + \theta_{P-1} x_n^{P-1}$$

$$\hat{y} = Xe$$

$$X \in \mathbb{R}^{n \times p}$$

$$X = \begin{bmatrix} 1 & x_0 & \cdots & x_0^{p-1} \\ 1 & x_1 & & x_1^{p-1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{n-1} & & x_{n-1}^{p-1} \end{bmatrix}$$

Design matrix X

Exercise 1

$y = Ax$ ← no y and x dependence

$A \in \mathbb{R}^{m \times n}$

$y \in \mathbb{R}^m \wedge x \in \mathbb{R}^n$

$$y_i = \sum_{j=1}^{n} a_{ij} x_j$$

$$\frac{\partial y_i}{\partial x_k} = a_{ik} \quad \text{for all } k$$

$i = 1, \ldots, m$

$j = 1, \ldots, n$

$$\frac{\partial \vec{y}}{\partial \vec{x}} = A$$

$$\frac{\partial \vec{y}}{\partial \vec{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots \\ \frac{\partial y_2}{\partial x_1} & \vdots & \\ \vdots & & \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \end{bmatrix}$$

Define a scalar

$$\alpha = y^T A x$$

$y \in \mathbb{R}^m \qquad x \in \mathbb{R}^n$

$A \in \mathbb{R}^{m \times n}$

$$\frac{\partial \alpha}{\partial \vec{y}} \quad \wedge \quad \frac{\partial \alpha}{\partial \vec{x}}$$

Define $\quad w^T = y^T A$

$$\alpha = \vec{w}^T \vec{x}$$

$$= \sum_i w_i x_i$$

$$\frac{\partial \alpha}{\partial x_k} = w_k \Rightarrow$$

$$\frac{\partial \alpha}{\partial \vec{x}} = \vec{w}^T = \vec{y}^T A$$

α is scalar :

$$\alpha = \alpha^T = x^T A^T y$$

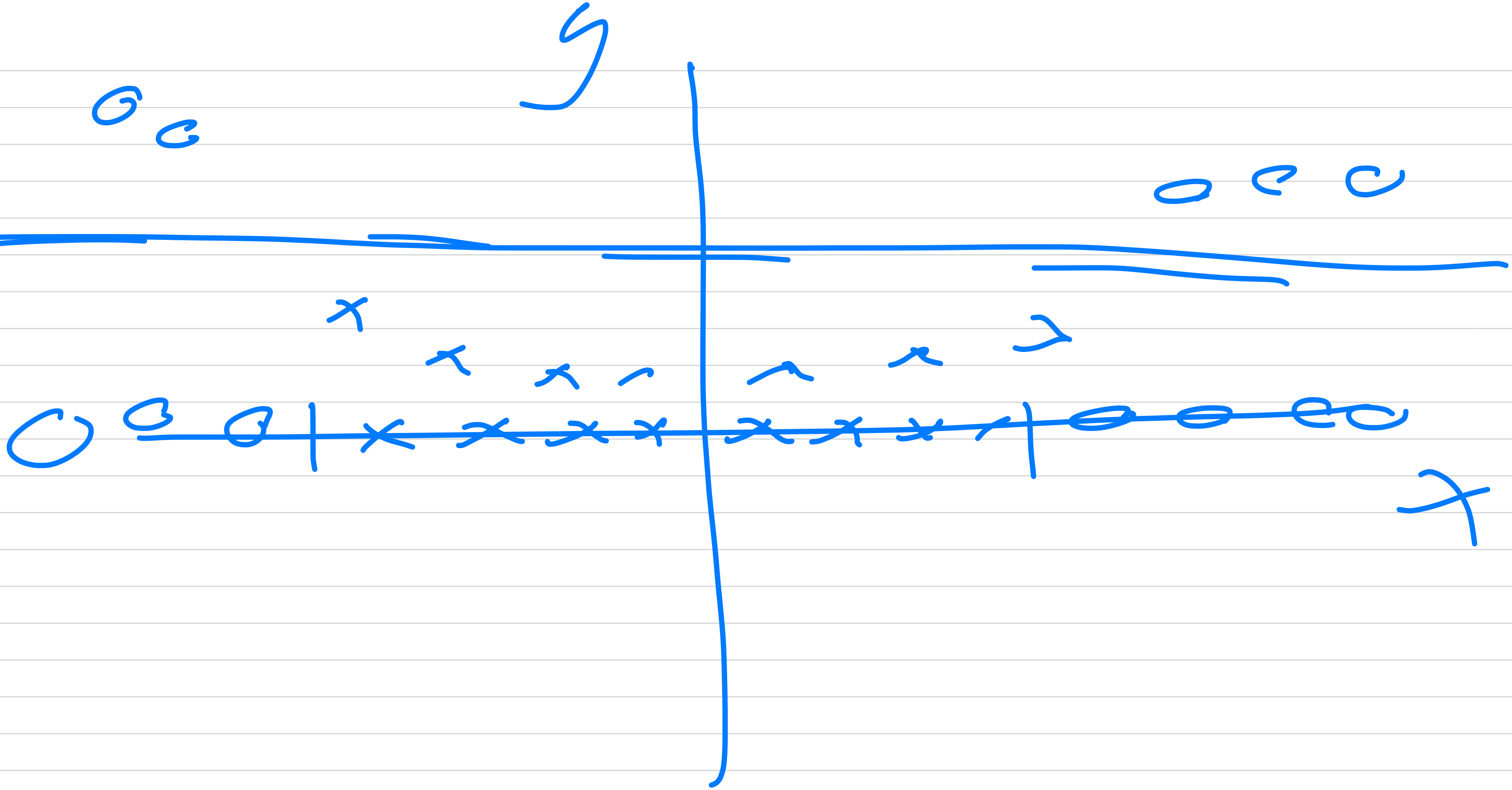
$$\frac{\partial \alpha}{\partial \vec{y}} = \overline{\vec{x}^T A^T}$$

$$\tilde{y}_{predict}(train) = X_{train}\,\hat{e}$$

$$\tilde{y}_{predict}(test) = X_{test}\,\hat{e}$$

$$\hat{e} = \left(X_{train}^T X_{train}\right)^{-1} X_{train}^T$$

$$\times\ y_{train}$$


y
x