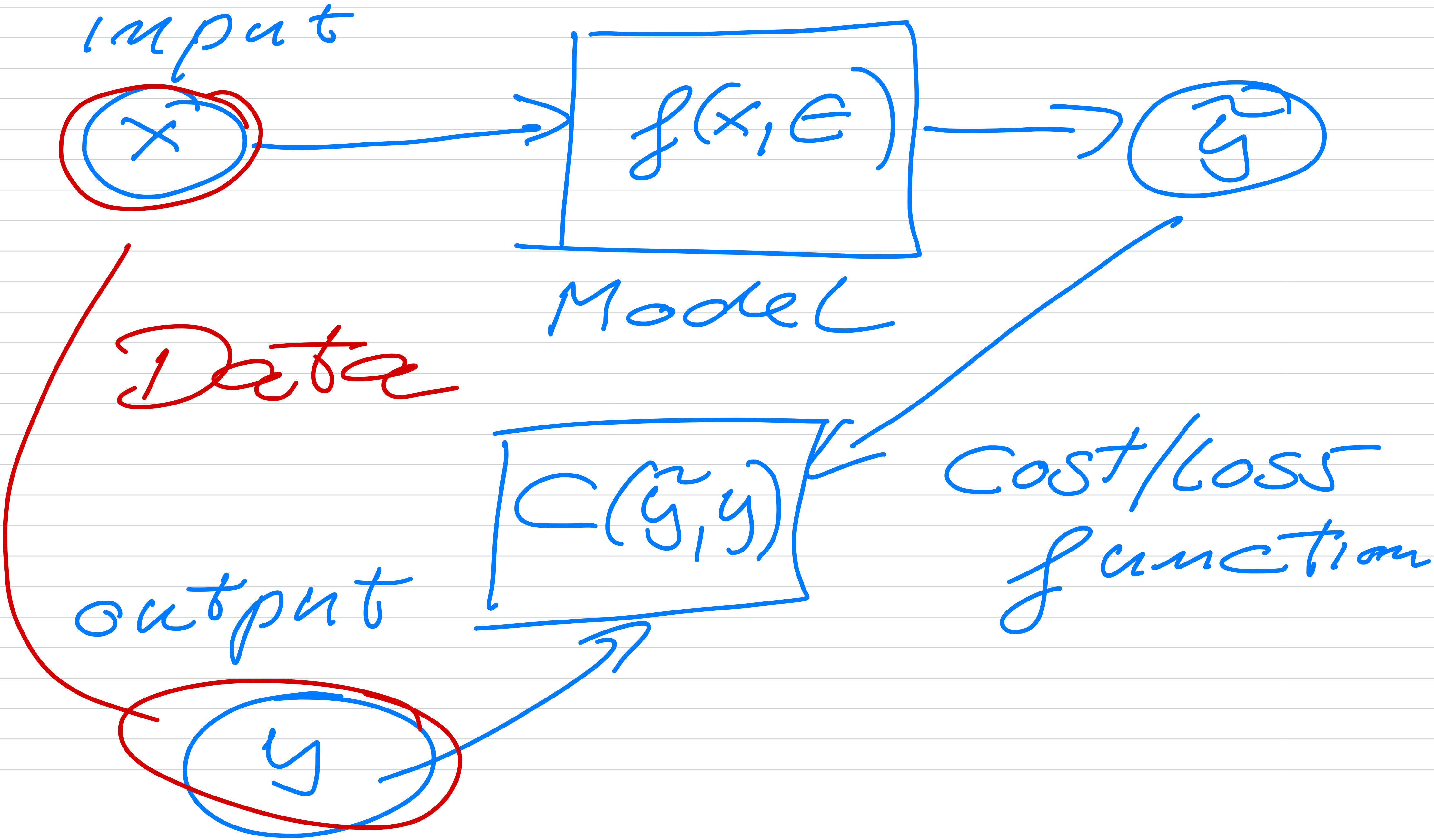
FYS-STK3155/4155 August 18, 2025



Mean squared error

$$C(y, \tilde{y}) = \boxed{\frac{1}{n} \sum_{i=0}^{n-1} (y_i - \tilde{y}_i)^2}$$

$$y \in \mathbb{R}^n$$

$$\tilde{y} \in \mathbb{R}^n$$

$$y^T = [y_0, y_1, \ldots\ldots y_{n-1}]$$

$$\tilde{y}^T = [\tilde{y}_0, \tilde{y}_1, \ldots\ldots \tilde{y}_{n-1}]$$

optimization

$$\frac{\partial C}{\partial \theta} = 0$$

$$|y - \hat{y}|$$

$$\frac{d|x|}{dx} = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

$$(x, y)$$

$$\hat{y} \simeq y$$

$$\hat{y} = \alpha + \beta x$$

slope

intercept

$$\theta = \{\alpha, \beta\}$$

$$\theta^T = [\alpha, \beta] \quad p = 2$$

$$\frac{\partial c}{\partial \alpha} = 0 \wedge \frac{\partial c}{\partial \beta} = 0$$

$$\hat{\theta} = \underset{\theta \in \mathbb{R}^p}{\arg\min}\, C(y, \tilde{y})$$

optimal value

$$\tilde{y}_0 = \alpha + \beta x_0$$

$$\tilde{y}_1 = \alpha + \beta x_1$$

$$\vdots$$

$$X \in \mathbb{R}^{n \times 2}$$

$$\tilde{y}_{n-1} = \alpha + \beta x_{n-1}$$

Design matrix (feature)

$$X = \begin{bmatrix} 1 & x_0 \\ 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{n-1} \end{bmatrix}$$

$$\tilde{y} \in \mathbb{R}^{n}$$

$$X \in \mathbb{R}^{n \times 2} \quad \left(\mathbb{R}^{n \times p}\right)$$

$$\tilde{y} = X\theta \qquad \theta^T = [\alpha \ \beta]$$

$$C(y, \tilde{y}) = \frac{1}{n} \sum_i (y_i - \tilde{y}_i)^2$$

$$= \frac{1}{n} \sum_i (y_i - \alpha - \beta x_i)^2$$

$$C(y, \hat{y}) = \frac{1}{n}(y - X\theta)^T$$

$$X(y - X\theta)$$

$$\frac{\partial C}{\partial \theta} = 0 = X^T(y - X\theta)$$

$$\Rightarrow \hat{\theta} = \frac{1}{\overline{X^T X}} X^T y$$